Applecross Senior High School | Name :___________________

Mathematics Method 3 Test 1, 2017 | Time Allowed : 55 minutes.

**Note : For both Section 1 and section 2, working out must be shown for full marks to be awarded.**

**Section 1 : [ / 17 marks ]** **Section 2 : [ / 31 marks ]** **Total : [ / 48 marks ] = _____ %**

**Section 1 : Calculator and Resource Free** **Time : 20 minutes**

1. [ 3,2 = 5 marks ]

Differentiate the following with respect to x.

a) $f(x) = \frac{-x}{x^2+1}$ { Express numerator in simplest form }

b) $y = (1-x)^3 \left(1 + \frac{2}{x}\right)^2$ { Apply the product rule but do not simplify }

2. [2,2= 4 marks ]

A particle moves in a straight line such that its velocity, v m/s, depends upon displacement , x m , from some fixed point O according to the rule $v = 5x-4$

a) Find an expression in terms of x for the acceleration of the particle.

b) Determine the displacement and the acceleration of the particle when $v = 6$m/s.

3. [ 8 marks ]

The equation of the tangent to the curve $y = ax^3 - bx^2 + 2$ where $x = -1$ is $y = 18x + c$.

The curve has a point of inflection when $x = 1$.

Find the values of a, b and c.

[Note : Working out must be shown]

Name : ______________________________ Marks: $\overline{\phantom{00}}$ 31

**Section 2 : Calculator and Resource Assumed.** **Time Allowed : 35 minutes**

**Note : Show working for full marks to be awarded.**

1. [ 2,2= 4 marks ]

   A company produces **n** items of a certain product.
   The cost function \$C is given by $C(n) = 1200 + 5n^{1/3}$
   Each item sells for \$52.

   Find

   a) An expression for the marginal profit $P'(n)$

   b) A value for $P'(64)$ and comment on its meaning.

2. [ 2,4,4 = 10 marks ]

a) It takes 12 hours to drain a storage tank by opening the valve at the bottom. The **depth 'y'** of fluid in the tank **'t' hours after** the valve is opened is given by

**$y = 6\left(1 - \frac{t}{12}\right)^2$ metres.**

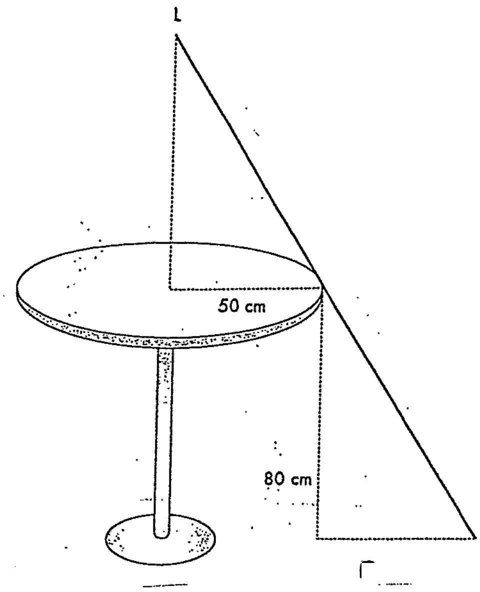
i) Show ,with full working out , the rate $\frac{dy}{dt}$ m/hour at which the tank is draining at time t is $\frac{t}{12} - 1$

ii) **a) When** is the fluid in the tank falling **fastest** and **slowest** ?

**b) What** are the values of $\frac{dy}{dt}$ at these times?

b) If the volume of a cylinder is given by $V = 2\pi r^3$ , find the approximate percentage change in V when r changes by $\frac{1}{2}$%.

3. [ 1,3 = 4 marks]




A table has a radius of 50 cm and a height of 80 cm.

A light ( L ) is lowered vertically downwards from a point above the centre of the table at a constant rate of 0.2 cm per second.

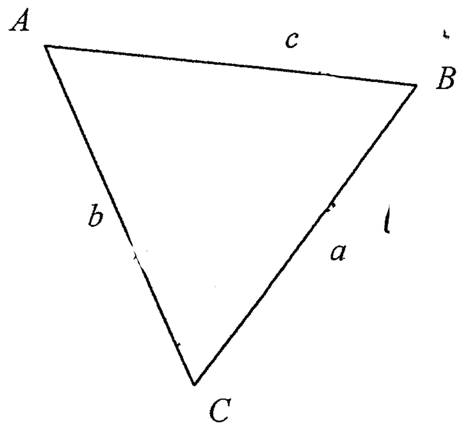
When the light is h cm above the table it casts a shadow that extends r cm from the edge of the table.

a) Show that $r = \frac{4000}{h}$

b) Find the rate at which r is changing when h = 60

4. [ 5 marks ]

The area of a triangle can be found by the formula : Area $= \frac{ab \, Sin \, C}{2}$



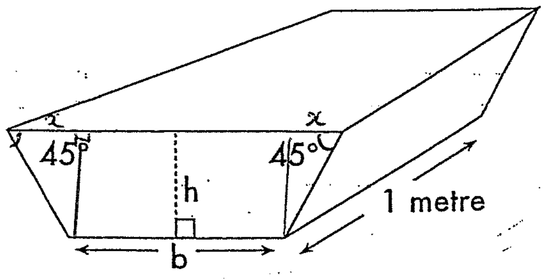

Using the **incremental formula**, determine the approximate change in area ( to 3 decimal places) of **an equilateral triangle** with each side of 10 cm, when each side increases by 0.1cm.

[ Hint : Use exact value for $60^0$ ]

5. [3,2,3 = 8 marks]

An animal drinking trough is constructed from stainless steel in the shape of a trapezoidal prism, with height 'h' metres and length of 1 metre.

The cross section of the prism is an isosceles trapezium with acute angle of $45^0$, base 'b' metres and area of 60 $m^2$.




a) Show that $b = \frac{60}{h} - h$

b) Show that the surface area 'A' in $m^2$ is : $A = \frac{60}{h} - h + 2h\sqrt{2} + 120$

c) Find the depth of the drinking trough to the nearest mm, if the amount of stainless steel is to be kept to a minimum. Justify your answer by using Calculus techniques.

End of Test